Lines and Planes

Problem 1 (Stewart, Exercise 12.4.45)**.** *Let P be a point not on the line ℓ that passes through the point Q and R. Show that the distance d from the point P to the line ℓ is*

$$d = \frac{|a \times b|}{|a|},$$

where a and b are the vectos QR and QP, respectively. Use this formula to find the distance from $(1, 1, 1)$ to the line that passes through the points $(0, 6, 8)$ and $(-1, 4, 7)$.

Problem 2 (Stewart, Exercise 12.5.70)**.** *Find the distance from the point $(0, 1, 3)$ to the line*

$$\frac{x}{2} = \frac{6 - y}{2} = z - 3.$$

Problem 3 (Stewart, Exercises 12.5.(27,31, 37, 39))**.** *Find the equation of the plane:*

(1) through the point $(2, 0, 1)$ and parallel to the plane $5x - y - z = 6$;

(2) through the points $(0, 1, 1)$, $(1, 0, 1)$, and $(1, 1, 0)$;

(3) that passes through the points $(3, 1, 4)$ and contains the line of intersection of the planes $x + 2y + 3z = 1$ and $2x - y + z = -3$;

(4) that passes through the point $(1, 5, 1)$ and is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$.

Problem 4 (Stewart, Exercises 12.5.(41, 44))**.** *Sketch the plane*

(1) $2x + 5y + z = 10$;

(2) $3x + 2y = 12$;

(3) $z = 7x$;

(4) $6x + 5y - 3z = 15$.

Problem 5 (Stewart, Exercises 12.5.(75, 76))**.** *Show that the distance between the parallel planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$ is*

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

Find the equation of the planes that are parallel to the plane $x + 2y - 2z = 1$ and two units away from it.

Problem 6 (Stewart, Exercise 12.5.77)**.** *Show that the lines with symmetric equations $x = y = z$ and $x + 1 = y/2 = z/3$ are skew. Find the distance between these lines.*

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.